

VJEŽBA 6

1. Koliko iznose procentualne sigurne granice grešaka serijske, odnosno paralelne veze dva kondenzatora.

$$C_1 = 0.1 \mu F \pm 1\%$$

$$C_2 = 2 \mu F \pm 1.5\%$$

Kapacitet serijske veze je:

$$C_s = \frac{C_1 C_2}{C_1 + C_2}$$

Apsolutni iznos sigurnih granica grešaka računamo:

$$G_C = \pm \sum_{i=1}^n \left| \frac{\partial C_s}{\partial C_i} G_i \right|$$

$$G_C = \pm \left[\left| \frac{\partial C_s}{\partial C_1} G_1 \right| + \left| \frac{\partial C_s}{\partial C_2} G_2 \right| \right] \quad \text{pri čemu je } \frac{\partial C_s}{\partial C_1} = \frac{C_2^2}{(C_1 + C_2)^2} \quad \text{i} \quad \frac{\partial C_s}{\partial C_2} = \frac{C_1^2}{(C_1 + C_2)^2}$$

$$G_C = \pm \left[\left| \frac{C_2^2}{(C_1 + C_2)^2} G_1 \right| + \left| \frac{C_1^2}{(C_1 + C_2)^2} G_2 \right| \right]$$

$$G_{1\%} = \frac{G_1}{C_1} 100 \rightarrow G_1 = \frac{G_{1\%} C_1}{100} \quad \text{i} \quad G_{2\%} = \frac{G_2}{C_2} 100 \rightarrow G_2 = \frac{G_{2\%} C_2}{100} \quad \text{pa je}$$

$$G_C = \pm \left[\left| \frac{C_2^2}{(C_1 + C_2)^2} \frac{G_{1\%} C_1}{100} \right| + \left| \frac{C_1^2}{(C_1 + C_2)^2} \frac{G_{2\%} C_2}{100} \right| \right]$$

Procentualna sigurna granica grešaka serijske veze je:

$$G_{C\%} = \frac{G_C}{C_s} 100 = \pm \left[\left| \frac{C_2}{(C_1 + C_2)} G_{1\%} \right| + \left| \frac{C_1}{(C_1 + C_2)} G_{2\%} \right| \right] = \pm 1.024\%$$

Kapacitet paralelne veze je:

$$C_p = C_1 + C_2$$

$$G_C = \pm \left[\left| \frac{\partial C_s}{\partial C_1} G_1 \right| + \left| \frac{\partial C_s}{\partial C_2} G_2 \right| \right] \text{ a } \frac{\partial C_p}{\partial C_1} = 1 \text{ i } \frac{\partial C_p}{\partial C_2} = 1$$

$$G_C = \pm \left[|G_1| + |G_2| \right] = \pm \left[\left| \frac{G_{1\%} C_1}{100} \right| + \left| \frac{G_{2\%} C_2}{100} \right| \right]$$

$$G_{C\%} = \frac{G_C}{C_s} 100 = \pm \left[\left| \frac{C_1}{(C_1 + C_2)} G_{1\%} \right| + \left| \frac{C_2}{(C_1 + C_2)^2} G_{2\%} \right| \right] = \pm 1.07\%$$

2. Sa kojom tačnošću je izmjerena potrošnja električne energije W jednosmjerne struje I u toku vremena t na potrošaču otpora R ako je greška mjerenja struje 2%, greška mjerenja otpora 1% i greška mjerenja vremena 0.5%.

Električna energija potrošača jednosmjerne struje data je relacijom:

$$W = RI^2t$$

$$G = \pm \left[\left| \frac{\partial W}{\partial I} G_I \right| + \left| \frac{\partial W}{\partial R} G_R \right| + \left| \frac{\partial W}{\partial t} G_t \right| \right]$$

$$\frac{\partial W}{\partial I} = 2RI t$$

$$\frac{\partial W}{\partial R} = I^2 t$$

$$\frac{\partial W}{\partial t} = RI^2$$

$$G = \pm \left[|2RI t G_I| + |I^2 t G_R| + |RI^2 G_t| \right]$$

$$G_{\%} = \frac{G}{W} 100 = \pm \left[\left| \frac{2RI t}{W} G_I 100 \right| + \left| \frac{I^2 t}{W} G_R 100 \right| + \left| \frac{RI^2}{W} G_t 100 \right| \right]$$

$$G_{\%} = \frac{G}{W} 100 = \pm \left[\left| \frac{2RI t}{RI^2 t} G_I 100 \right| + \left| \frac{I^2 t}{RI^2 t} G_R 100 \right| + \left| \frac{RI^2}{RI^2 t} G_t 100 \right| \right]$$

$$G_{\%} = \frac{G}{W} 100 = \pm \left[\left| \frac{2}{I} G_I 100 \right| + \left| \frac{1}{R} G_R 100 \right| + \left| \frac{1}{t} G_t 100 \right| \right]$$

$$G_{\%} = \frac{G}{W} 100 = \pm \left[|2G_{I\%}| + |G_{R\%}| + |G_{t\%}| \right] = 2 * 2\% + 1\% + 0.5\% = 5.5\%$$

3. Voltvetrom klase tačnosti 0.1 mjernog dometa 1V i unutrašnjeg otpora 5kΩ treba ostvariti mjerno područje 100V. Kojoj klasi tačnosti pripada voltvetar sa proširenim mjernim područjem ako su unutrašnji otpor voltmetra i predotpor sa granicama grešaka $\pm 0.2\%$

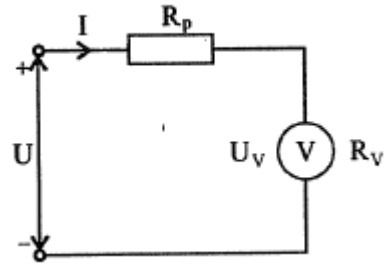
$$U=100V$$

$$U_V=1V$$

$$R_V=5k\Omega$$

$$k_t=0.1$$

$$U - IR_p - U_V = 0$$



$$U - \frac{U_V}{R_V} R_p - U_V = 0 \rightarrow U = U_V \left(\frac{R_p}{R_V} + 1 \right)$$

$$\rightarrow R_p = \left(\frac{U}{U_V} - 1 \right) R_V = 495k\Omega$$

$$\text{Sigurne granice greška iznose: } G = \pm \left[\left| \frac{\partial U}{\partial U_V} G_{U_V} \right| + \left| \frac{\partial U}{\partial R_V} G_{R_V} \right| + \left| \frac{\partial U}{\partial R_p} G_{R_p} \right| \right]$$

$$G = \pm \left[\left| \left(\frac{R_p}{R_V} + 1 \right) G_{U_V} \right| + \left| -U_V \frac{R_p}{R_V^2} G_{R_V} \right| + \left| \frac{U_V}{R_V} G_{R_p} \right| \right]$$

Procentualne granice greška iznose:

$$G_{\%} = \pm \frac{G}{U} 100 = \pm \left[\left| G_{U_V\%} \right| + \left| \frac{R_p}{R_p + R_V} G_{R_V\%} \right| + \left| \frac{R_p}{R_p + R_V} G_{R_p\%} \right| \right] =$$

$$\pm [0.1\% + 0.99 * 0.2\% + 0.99 * 0.2\%] = \pm 0.496\%$$

4. Koliko iznose sigurne granice greška otpora od 89.7Ω realizovanih sa kutijom otpornika sa čepovima (serijska kombinacija otpornika od 50, 20, 10, 10, 5, 2, 1, 1, 0.5, 0.2, 0.1, 0.1, 0.1 Ω) ako su granice greška ugrađenih otpornika $\pm 0.05\%$? Zanemariti kontaktne otpore.

Otpor od 89.7Ω se može realizovati kao $50+20+10+5+2+1+1+0.5+0.2=89.7$

Sigurne granice greška su:

$$G = \pm \left[\left| \frac{\partial R}{\partial R_1} G_{R_1} \right| + \left| \frac{\partial R}{\partial R_2} G_{R_2} \right| + \dots + \left| \frac{\partial R}{\partial R_9} G_{R_9} \right| \right]$$

Obzirom da su svi izvodi u ovom slučaju jednaki i iznose 1, sigurne granice greška možemo pisati u obliku:

$$G = \pm \left[\left| G_{R_1} \right| + \left| G_{R_2} \right| + \dots + \left| G_{R_9} \right| \right]$$

Sada, procentualne granice grešaka su:

$$G_{\%} = \frac{G}{R} 100 = \frac{\pm[|G_{R_1}| + \|G_{R_2}\| + \dots + \|G_{R_9}\|]}{R_1 + R_2 + \dots R_9} 100$$

$$G_{\%} = \frac{G}{R} 100 = \frac{\pm\left[\left|\frac{R_1 G_{R_1\%}}{100}\right| + \left\|\frac{R_2 G_{R_2\%}}{100}\right\| + \dots + \left\|\frac{R_9 G_{R_9\%}}{100}\right\|\right]}{R_1 + R_2 + \dots R_9} 100$$

Obzirom da je u postavci dato da je $G_{R_9\%} = G_{R_1\%} = G_{R_2\%} = G_{R_3\%} = \dots = G_{R_9\%} = \pm 0.05\%$

$$G_{\%} = 9G_{R_9\%} = \pm 0.45\%$$

Apsolutna vrijednost sigurnih granica grešaka je:

$$G = \frac{G_{\%}}{100} R = 0.4\Omega$$